

## Effects of Nozzle Amplitude on Production of Uniformly Sized Liquid Droplets

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This paper describes the effects of nozzle amplitude on production of uniformly sized liquid droplets using the longitudinal vibration of a nozzle. Uniformly sized liquid droplets could make complex phenomena of liquid atomization simple, so they could be useful for the fundamental studies such as combustions of liquid droplets as well as atomization. However, the effects of nozzle amplitude on their production which must be an important factor is not clarified. This experiment was carried out to investigate the effects of nozzle amplitude on the production of uniformly sized liquid droplets in detail.

### 1. Introduction

The uniformly sized liquid droplets can be utilized for the fundamental studies of coalescences, breakups, evaporations, and combustions of liquid droplets, so the development of techniques for producing uniformly sized liquid droplets is technologically important. The uniformly sized liquid droplets have been produced using vibration energies, centrifugal forces and electric powers. In particular, the production method by longitudinal vibration of a nozzle is superior to the others from the view point of simplicity of a device, uniformity of droplet diameters, and production rate of liquid droplets. Schneider and Hendricks [1] conducted the experiment using the vibration method, and reported that the range of the wavelength for producing uniformly sized liquid droplets,  $\lambda$ , was  $3.5d_j$  to  $7d_j$ , ( $d_j$  is a jet diameter) but did not comment on the effects of nozzle amplitude on their production. Dabora [2] made use of Rayleigh's criterion [3] for the breakup of capillary jets and succeeded in producing uniformly sized liquid droplets by setting the nozzle frequency to satisfy the most unstable condition of Rayleigh,  $f = u_j/4.508d_j$ . If the breakup condition of a liquid jet by the vibration of a nozzle coincides with Rayleigh's analysis, the condition  $\lambda > \pi d_j$ , is the necessary condition for producing uniformly sized liquid droplets. Rayleigh's analysis is constructed under

the supposition of small initial disturbances of a jet, thus if sufficiently large disturbances are applied to a jet as the initial disturbances, the condition for producing uniformly sized liquid droplets may differ from his theory. However, satisfactory systematic investigations of the effects of factors for producing uniformly sized liquid droplets have not been conducted. This experiment was carried out to investigate the effects of nozzle amplitude on production of uniformly sized liquid droplets [4].

### 2. Nomenclature

$a$  = jet radius  
 $A_n$  = nozzle amplitude  
 $d_d$  = droplet diameter  
 $d_j$  = jet diameter  
 $d_n$  = nozzle diameter  
 $f$  = nozzle frequency  
 $l_j$  = jet length  
 $q$  = liquid flow rate  
 $Re$  = Reynolds number ( $=u_j d_j/\nu$ )  
 $u_j$  = jet velocity  
 $\eta_e$  = jet amplitude at the nozzle exit  
 $\kappa$  = dimensionless frequency ( $=2\pi a f/u_j$ )  
 $\lambda_j$  = disturbance wavelength  
 $\mu$  = coefficient of viscosity  
 $\rho$  = liquid density  
 $\sigma$  = liquid surface tension

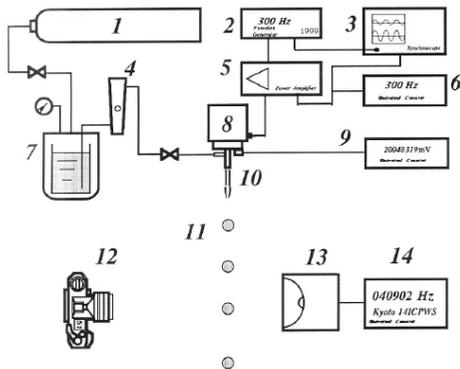
### 3. Experimental Apparatus and Condition

The schematic diagram of the experimental apparatus is shown in Fig. 1. The liquid in the tank is compressed by high-pressure air and supplied to the nozzle attached to the vibrator. The nozzle is vibrated longitudinally, controlling frequencies and amplitude using the oscillator and the power amplifier. In this way, the uniformly sized liquid droplets are produced using the treatment mentioned above. The production rate of liquid droplets is counted using the photo sensor and the universal counter. The amplitude of the nozzle is measured using the accelerometer, the digital multimeter and the synchroscope. The nozzle diameter is 0.732 mm. Water is used as the standard liquid, and 75 % (vol.) glycerin in water is used to investigate the effects of viscosity of liquids. The physical properties of water are noted as  $\sigma=74.7 \times 10^{-3}$  N/m,  $\rho=1.00 \times 10^3$  kg/m<sup>3</sup>,  $\mu=1.01 \times 10^{-3}$  Pa·s and those of 75 % (vol.) glycerin in water are noted as  $\sigma=70.2 \times 10^{-3}$  N/m,  $\rho=1.20 \times 10^3$  kg/m<sup>3</sup>,  $\mu=27.6 \times 10^{-3}$  Pa·s. The diameter of a liquid droplet is calculated with the law of mass conservation, using Eq. (1).

$$d_d = (6q/\pi f)^{1/3} \quad (1)$$

The jet wavelength given by the vibration of a nozzle is calculated using Eq. (2).

$$\lambda_j = u_j/f \quad (2)$$



1. compressed air, 2. function generator, 3. synchroscope, 4. water flow meter, 5. power amplifier, 6. universal counter, 7. liquid tank, 8. vibrator, 9. accelerometer, 10. nozzle, 11. liquid droplets, 12. camera, 13. stroboscope, 14. universal counter

Fig. 1. Experimental apparatus

### 4. Production of Uniformly Sized Liquid Droplets and Uniformity of Liquid Droplets

The typical phenomena of producing uniformly sized liquid droplets are shown in Fig. 2. In Fig. 2 (a), the liquid droplet diameters seem quite different, but in (b) they seem uniform. In fact, the liquid droplet diameters in (b) are uniform according to photographic measurement and the diameters are equal to numerical values. Sometimes a primary liquid droplet and a satellite liquid droplet coalesce, and they become one uniformly sized liquid droplet, but in this paper, this is not included as production of a uniformly sized liquid droplet. When short disturbance wavelengths are applied to a liquid jet, the uniformly sized liquid droplets coalesce with each other and as a result they become non-uniform. However, if they do not coalesce in the time to produce a few droplets, they are included as production of uniformly sized liquid droplet. The following phenomena can be observed when producing uniformly sized liquid droplets.

- (1) Production rate of liquid droplets is equal to the nozzle frequency.
- (2) Phenomena of producing uniformly sized liquid droplets can be observed in the same period using a stroboscope.
- (3) Jet length only varies by only one wavelength of a disturbance.

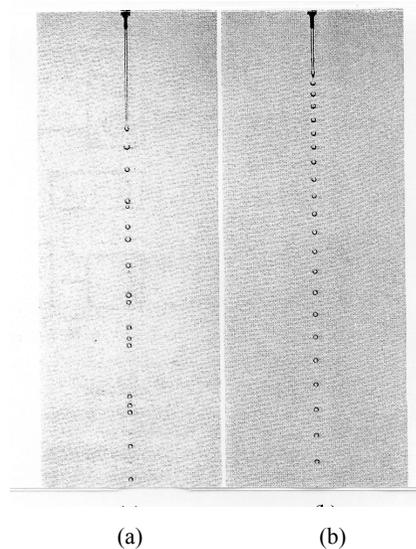


Fig. 2. Typical production of liquid droplets.

(4) Jet length decreases with an increasing amplitude of a nozzle.

**5. Effects of Nozzle Amplitude on the Jet Length**

Figure 3 is the photos showing the effects of nozzle amplitude on production of uniformly sized liquid droplets. The nozzle amplitude is varied under the constant jet velocity and the nozzle frequency. If vibration is not applied to the nozzle, the jet length varies irregularly and the diameters of the liquid droplets are not uniform. When the nozzle is vibrated longitudinally with amplitude of  $5.11 \mu\text{m}$ , the jet length decreases but the liquid droplet diameters are not yet uniform. When the nozzle is vibrated with amplitude of  $74.5$  and  $185 \mu\text{m}$ , the jet length decreases further and the liquid droplet diameters become uniform.

Figure 4 shows the relationship between the nozzle amplitude and the jet length of the water under the constant jet velocity. The experiment was conducted for four kinds of frequencies. At first, the jet length declines sharply with the increasing nozzle amplitude; however, later it decreases only

slightly. The nozzle frequency for the constant jet velocity, substantially the disturbance wavelength effects on the jet length. The nozzle amplitude is not so effective when the disturbance wavelengths are too much longer or shorter.

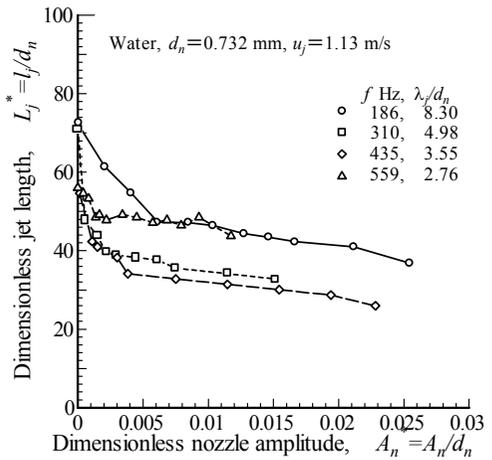


Fig. 4. Relationship between dimensionless nozzle amplitude and dimensionless jet length.

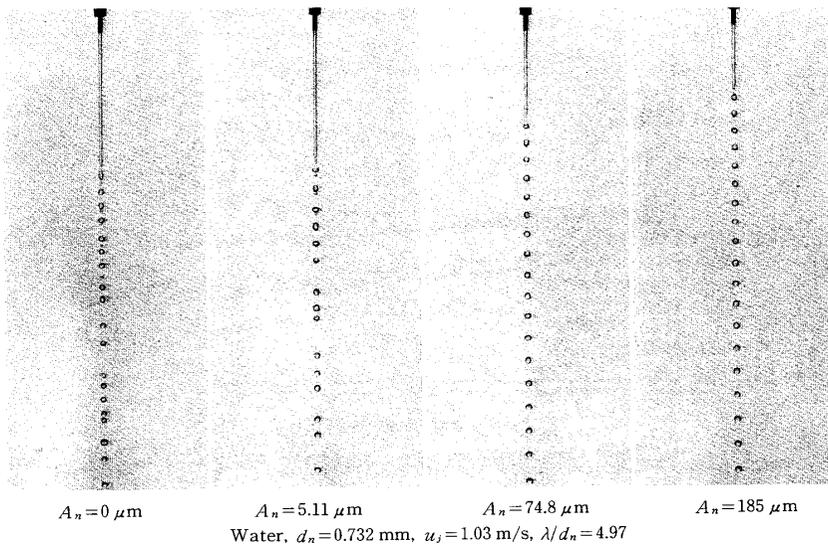


Fig. 3. Effects of nozzle amplitude for producing liquid droplets.

Figure 5 shows the relationship between the nozzle amplitude and the jet length of the 75 % (vol.) glycerin in water. As the kinematic viscosity is higher than water, the liquid column keeps laminar at higher velocity compare to water. So, the longer liquid column is formed. The jet length declines more sharply as the nozzle amplitude increase. This is the same tendency, but the sensitivity of the jet length for the 75 % (vol.) glycerin in water is higher compare to the water.

Figure 6 shows the comparison of the jet length of the water and the 75 % (vol.) glycerin in water for the same jet velocity. Under the condition of no vibration, the jet length of the 75% (vol.) glycerin in water to the nozzle amplitude is greater than that of the water. The nozzle amplitude required to produce uniformly sized droplets of glycerin in water is smaller than that of the water. The sensitivity of the jet length of the uniformly sized liquid droplets differs with each liquid. In the 75 % (vol.) glycerin in water, uniformly sized liquid droplets can be produced using smaller nozzle amplitude than the water.

Figure 7 shows the relationship between the nozzle frequency and the nozzle amplitude to produce uniformly sized liquid droplets. The liquid used is water. The lateral axis is the nozzle frequency, while the longitudinal axis is the nozzle amplitude. The uniformly sized liquid droplets can

be produced not only at the most unstable nozzle frequency of Rayleigh's analysis but also in a certain range of nozzle frequencies. In the middle range of nozzle frequencies, uniformly sized liquid droplets can be produced with smaller nozzle amplitude, but in the lower and the higher ranges, it needs larger nozzle amplitude to produce uniformly sized liquid droplets.

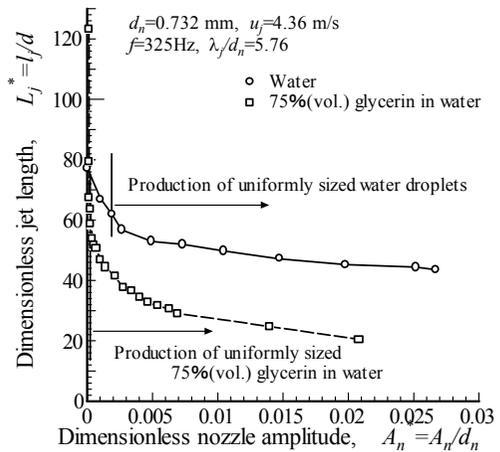


Fig. 6. Relationship between nozzle amplitude and jet length.

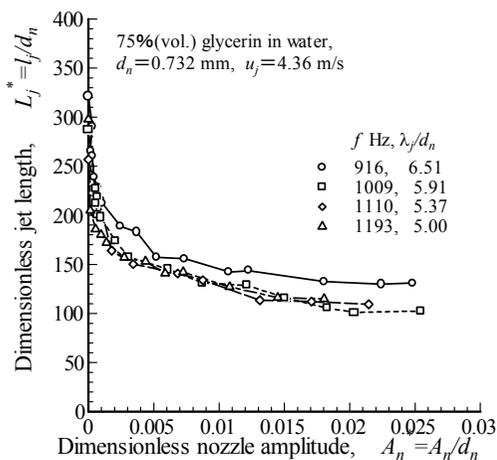


Fig. 5. Relationship between dimensionless nozzle amplitude and dimensionless jet length.

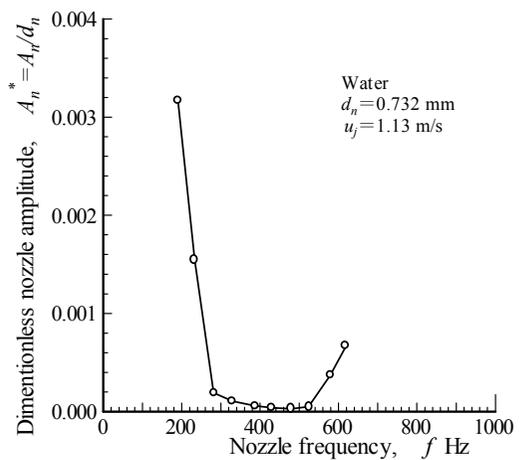


Fig. 7. Relationship between nozzle frequency and dimensionless nozzle amplitude.

**6. Discussion**

**6.1 Volume with Nozzle Amplitude for the Half Period of Nozzle Vibration**

The quantity of the liquid issued from the nozzle is periodically varied by the nozzle amplitude of the shape of a jet at the exit of a nozzle is also varied. The amplitude,  $\eta_e$ , of a jet at the nozzle exit is theoretically calculated under the supposition that the shape of a jet is a sinusoidal wave. It is supposed that the shape of a jet is axial symmetric, and the jet radius without nozzle amplitude is “ $a$ ” (Fig. 8). The increased volume,  $V'$ , with nozzle amplitude for the half period of nozzle vibration is calculated using Eq. (3).

$$V' = \pi \int_0^{\lambda_j/2} (a + \eta_e \sin \frac{2\pi}{\lambda_j} z) dz - \pi \int_0^{\lambda_j/2} a^2 dz = 2a\eta_e \lambda_j + \frac{\pi}{4} \eta_e^2 \lambda_j \tag{3}$$

$$\pi \lambda_j \eta_e^2 + 8a\lambda_j \eta_e - 4V' = 0 \tag{4}$$

Therefore,  $\eta_e$  can be calculated considering it positive, using Eq. (5).

$$\eta_e = \frac{-4a\lambda_j + 2\sqrt{\lambda_j(4a^2\lambda_j + \pi V')}}{\pi \lambda_j} \tag{5}$$

where,  $V' = 4\pi a^2 A_n |I|$ ,  $\lambda_j = u_j/f$ ,  $I = U/B$   
 $U = \{0.7071068 + 0.125/x + 0.0994368/x^2 + 0.076$   
 $1718/x^3 + 0.0064737/x^4 - i(0.7071068 -$   
 $0.375/x - 0.1657281/x^2 - 0.0878906/x^3 +$   
 $0.02718977/x^4)\}$  (6)

$$B = x(-i + 0.1767767/x + 0.015625/x^2 - 0.0911504/x^3 - 0.2193603/x^4)$$
 (7)

$$x = \sqrt{\kappa Re/2} = \sqrt{d_n^2 \pi \rho f / \mu} \tag{8}$$

It is noted that the amplitude of a jet at the exit of a nozzle increases with the increasing nozzle

amplitude and the viscosity of the liquid. It may be regarded that this is the reason why the jet length decreases with the increasing nozzle amplitude and why the sensitivity of a jet for nozzle amplitude depends on the viscosity of the liquids.

**6.2 Penetration Depth Due to Nozzle Vibration**

Viscosity for glycerin is greater than that of water. Viscosity plays an important role for the depth of penetration of the viscous wave in a vibrating nozzle. Shearing stress in liquids is influenced by viscosity and the velocity gradient.

Figure 9 shows the relationship between absolute of dimensionless velocity profiles of the disturbance flow and the dimension radius. The disturbance flow in the nozzle is caused due to the vibration of the nozzle. The symbol “ $r$ ” is the distance from the center of the nozzle axis, while “ $a$ ” means the nozzle radius. So, the dimensionless radius  $R^* = r/a = 0$  means the center axis of the nozzle, while  $R^* = 1$  means the wall of the nozzle. The velocity profiles of the maximum disturbance flow are shown for various parameters  $\kappa Re$ . As shown in Eq. (8),  $\kappa Re/2$  equals  $d_n^2 \pi \rho f / \mu$ . Under the condition of a certain nozzle diameter and liquid, the increase of  $\kappa Re$  means the increase of the nozzle frequency. So, as  $\kappa Re$  increases, that is, as the nozzle frequency increases, the effects of the nozzle amplitude are limited near the nozzle wall, and the maximum velocity of the disturbance flow at the center axis decreases. That is, as the nozzle frequency increases, the penetration depth decreases. In contract, under the certain nozzle frequency, as the viscosity of the liquid increases, the penetration depth increases. The density of liquid is different for each liquid; however, its value is not so different compare to the

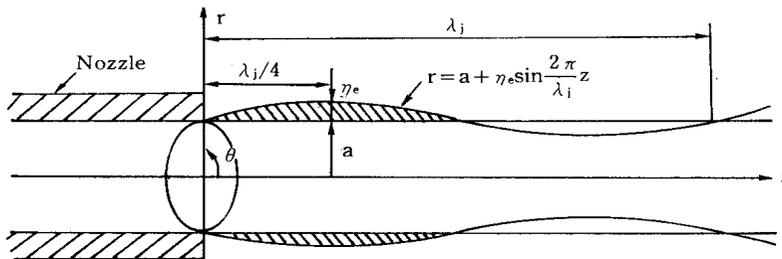


Fig. 8. Jet figure around a nozzle exit.

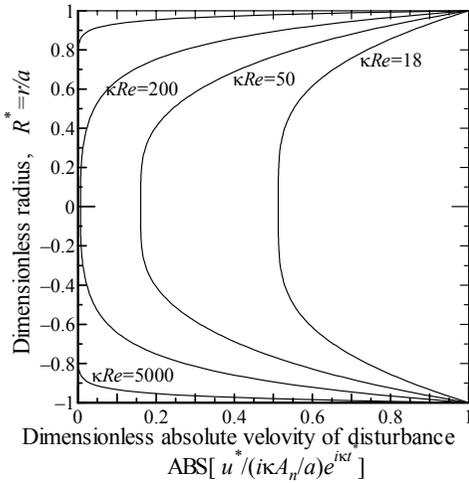


Fig. 9. Dimensionless velocity profiles.

viscosity.

When the nozzle radius “a” is constant,  $\kappa Re$  substantially means  $f/\nu$ . This means that the penetration depth increases as the viscosity increases. Therefore, the nozzle amplitude is more effective for 75 %(vol.) glycerin in water than the water.

The velocity profiles of the disturbance flow shown in Fig. 9 are the maximum velocities. When the nozzle reaches to the highest/lowest position of the longitudinal vibration stroke, it stops its movement. When the nozzle stops, the disturbance flow velocity of a liquid at the nozzle wall is 0; however, the disturbance flow velocity apart from the nozzle wall is not 0. That is, the phase lag of the disturbance flow velocity occurs. This phase lag also depends on the  $\kappa Re$ . As the liquid viscosity increases, the phase lag decreases.

### 7. Conclusions

A study on the effects of nozzle amplitude on production of uniformly sized liquid droplets by vibrating a nozzle was carried out. As a result, the following conclusions were obtained.

- (1) Uniformly sized liquid droplets were produced under the condition of suitable wavelengths and nozzle amplitude.
- (2) The jet length decreased with the increasing nozzle amplitude.

- (3) The sensitivity of a liquid jet for nozzle amplitude increased as the viscosity of the liquids increased.
- (4) The production rate of uniformly sized liquid droplets coincided with the nozzle frequency.

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