

A Comprehensive Gibbs Thermodynamic Potential of Ice

Rainer Feistel^{1*} and Wolfgang Wagner²

¹Institut für Ostseeforschung an der Universität Rostock, D-18119 Warnemünde, Germany

*E-mail: rainer.feistel@io-warnemuende.de

²Lehrstuhl für Thermodynamik, Ruhr-Universität Bochum, D-44780 Bochum, Germany

Various thermodynamic equilibrium properties of naturally abundant, hexagonal ice (ice Ih), measured during the past 100 years, have been used to develop a Gibbs energy function $g(T, P)$ of temperature and pressure, covering the ranges 0-273.16 K and 0.2 μ Pa - 210 MPa, expressed in the temperature scale ITS-90. This new equation extends the recently developed Gibbs functions to very low temperatures and high pressures, including e.g. Debye's cubic temperature law of heat capacity and Pauling's residual entropy, and is based on a significantly enlarged experimental data set compared to its predecessors. It provides a physical, absolute entropy reference for liquid water at the triple point. Due to the inherent thermodynamic cross-relations, the formulas for particular quantities like density, thermal expansion or compressibility have thus become fully consistent with each other, are more reliable now, and extended in their ranges of validity. As an example, the actual assembly of data results in a Clausius-Clapeyron slope of the melting line at the triple point with significantly reduced error. In conjunction with the IAPWS-95 formulation for the fluid phases of water, the new Gibbs energy function, i.e. the chemical potential of ice, allows an alternative computation of the melting and sublimation curves, being improved especially near the triple point. Combined with the Gibbs function of seawater, the Gibbs function of ice provides a suitable thermodynamic description of sea ice over the oceanographic, so-called 'Neptunian' ranges of pressures, temperatures, and salinities.

1. Introduction

The latest development of more comprehensive and more accurate formulae for thermodynamic equilibrium properties of seawater in form of a Gibbs potential function [1,2] was based on the current scientific pure-water standard IAPWS-95 [3]. For an adequately advanced description of freezing points of seawater over the natural, 'Neptunian' ranges of salinity and pressure, for the consistent description of saturated vapour pressures over ice and sea ice, as well as for an improved Gibbs potential formulation of sea ice thermodynamics, the development of a reliable Gibbs function of naturally abundant hexagonal ice Ih is desired, valid over a wide range of pressures and temperatures. Such a function is presented in this paper; the detailed derivation of its functional form and many details of the fitting procedures are given in [4].

After the extensive and systematic laboratory measurements of ice Ih and other solid aqueous phases by Bridgman [5-8], various reviews on ice

and sea ice properties and comprehensive presentations thereof were published later, e.g. [9-20]. It is still uncertain whether or not ice Ih possesses a density maximum at a finite temperature above 0 K. The Gibbs function derived here, in agreement with experimental findings of e.g. Brill and Tippe [21], does not exhibit negative thermal expansion coefficients, although more recent numerical model calculations by Tanaka [22] and measurements by Röttger et al. [23] show a density maximum at about 60-70 K.

The first proposals to combine ice properties into its Gibbs function were published by Feistel and Hagen [24], and by Tillner-Roth [25]. Both formulae provide specific free enthalpy of ice, $g(T, P)$, in terms of temperature T and pressure P , and are based on only restricted data selections from the vicinity of the melting curve. Feistel and Hagen [24] had used ice properties as summarized by Yen et al. [17], expressed in lowest order polynomials of temperature and pressure near the melting point at normal pressure, later improved by Feistel [1] for higher pressures using the melting point equation

Table 1. Special constants and values used in the paper. P_t : Triple point pressure, P_0 : Normal pressure, T_t : Triple point temperature, T_0 : Celsius zero point, T_m : Normal pressure melting point, T_M : Improved normal pressure melting point, this paper

quant	value	unit	abs. error	ref.
P_t	611.657	Pa	0.010	[48]
P_0	101325	Pa	exact	[49]
T_t	273.160	K	exact	[50]
T_0	273.150	K	exact	[50]
T_m	273.153	K	0.001	[3]
T_M	273.152518	K	2×10^{-6}	

of Wagner et al. [26]. Tillner-Roth [25] used the latter equation together with selected ice properties along the entire melting curve up to the triple point ice I-III-liquid, which is at about 210 MPa and -22°C .

It is the aim of this paper to improve the previously existing Gibbs functions of ice by additionally including more suitable, theoretical as well as measured, available ice properties, covering its entire existence region in the pressure-temperature diagram. The function proposed here can be used for constructing an equation of state of sea ice over the full natural pressure range of the oceans, including a freezing point formula. No experimental data are available for the region of high pressures at low temperatures. The new Gibbs potential provides reasonable values for that area, but no error estimates can be given. All temperature values of the measurements used have been converted to the ITS-90 temperature scale. A list of some general constants and values is given in Table 1 for reference.

2. Gibbs Potential Function

The thermodynamic Gibbs potential function $g(T, P)$ is specific free enthalpy (specific Gibbs energy) of ice. We express absolute temperature T by the dimensionless variable $t = T/T_t$ with triple point temperature T_t and absolute pressure P by $p = P/P_t$ with triple point pressure P_t .

The functional form of $g(T, P)$ for ice Ih is given by eqs. (1) and (2) as the real part of a complex function of temperature,

$$\frac{g(T, P)}{g_U} = g_0 - s_0 \tau \cdot t + \tau \operatorname{Re} \sum_{k=1}^2 r_k \left[(t_k - t) \ln(t_k - t) - 2t_k \ln t_k + (t_k + t) \ln(t_k + t) - \frac{t^2}{t_k} \right] \quad (1)$$

with two coefficients being polynomials of pressure,

$$g_0(p) = \sum_{k=0}^4 g_{0k} \cdot (p - p_0)^k, \quad r_2(p) = \sum_{k=0}^2 r_{2k} \cdot (p - p_0)^k. \quad (2)$$

The unit specific free enthalpy is $g_U = 1\text{J/kg}$, the dimensionless constant τ is $\tau = \frac{T_t}{1\text{K}} = 273.160$ and the dimensionless normal pressure is $p_0 = \frac{P_0}{P_t} = 165.657$. The real constants g_{00} to g_{04} and s_0 as well as the complex constants t_1, r_1, t_2 and r_{20} to r_{22} are given in Table 2.

Table 2. Coefficients of the Gibbs function as given in eqs. (1), (2). The residual entropy coefficient s_0 is given in two versions, “absolute” and “IAPWS-95”, see text.

coeff	value
g_{00}	-632578.704355102
g_{01}	0.655029997804786
g_{02}	-1.89952376891314E-08
g_{03}	3.40692612753936E-15
g_{04}	-5.78593658679522E-22
s_0 (IAPWS-95)	-3333.18160308627
s_0 (absolute)	189.13
$\operatorname{Re} t_1$	3.71539090346389E-02
$\operatorname{Im} t_1$	5.10464771184122E-02
$\operatorname{Re} r_1$	45.951447199735
$\operatorname{Im} r_1$	65.223705014775
$\operatorname{Re} t_2$	0.345095829562823
$\operatorname{Im} t_2$	0.343315892017841
$\operatorname{Re} r_{20}$	-75.8695106343435
$\operatorname{Im} r_{20}$	-80.9878506462645
$\operatorname{Re} r_{21}$	-5.75529765634353E-05
$\operatorname{Im} r_{21}$	5.09059011946526E-05
$\operatorname{Re} r_{22}$	2.39617513518116E-11
$\operatorname{Im} r_{22}$	-2.73297877749166E-11

The complex logarithm $\ln(z)$ is the principal value, i.e. it evaluates to imaginary parts in the interval $-\pi < \text{Im} \ln(z) \leq +\pi$. The complex notation used here has no direct physical reasons but serves for convenience of analytical partial derivatives and for compactness of the resulting formulae. The residual entropy coefficient s_0 is given in Table 2 in form of two alternative values, its ‘IAPWS-95’ version is required for phase equilibria studies between ice and pure water in the IAPWS-95 formulation [3], or seawater [1], while its ‘absolute’ version represents the true physical zero-point entropy of ice [27, 28].

3. Regression Summary

The Gibbs potential function (1) has a compact mathematical structure which is capable of covering the entire range of existence of ice Ih between 0 and 273.16 K and 0 and 211 MPa. It uses 16 free parameters; 14 of them were determined by regression with respect to 339 data points belonging to 26 different groups of measurements (Table 3), the remaining 2 parameters are subject to the IAPWS-95 definition of internal energy and entropy of liquid water at the triple point, or alternatively, to the physically determined zero point residual entropy. The list of 18 parameters given in Table 2 contains 2 redundant ones which formally appeared during the transformation of 6 real parameters describing heat capacity into 4 complex numbers [4]. The majority of the various measured thermodynamic equilibrium properties are described by the formulation within their experimental uncertainties. Additionally, the cubic law of Debye for the heat

Table 3. Summary of data used for the determination of the Gibbs function coefficients. “Quantity” are: g : specific Gibbs energy, dP/dT : Clausius-Clapeyron slope of the melting curve, $s = -(\partial g / \partial T)_P$: specific entropy, $c_p = -T(\partial s / \partial T)_P$: isobaric specific heat capacity, $v = (\partial g / \partial P)_T$: specific volume, $\kappa_s = -(1/v)(\partial v / \partial P)_s$: isentropic compressibility. “Points” is the number of data points used with source, “ T ” and “ P ” are the intervals in temperature and pressure covered, “required r.m.s.” is error tolerance for the fit (essentially the experimental accuracy), “resulting r.m.s.” is the returned root mean square deviation after the fit. 1 TPa equals 10^{12} Pa.

quantity points	T /K P /MPa	required r.m.s. resulting r.m.s.
g 15 [5]	251-273 10-210	1150 J kg ⁻¹ 233 J kg ⁻¹
g 6 [29]	259-273 5-147	500 J kg ⁻¹ 48 J kg ⁻¹
g 45 [30]	257-273 0.0001-0.0006	139 J kg ⁻¹ 132 J kg ⁻¹
g 6 [31]	257-273 0.0001-0.0006	86 J kg ⁻¹ 175 J kg ⁻¹
dP/dT 1 [32]	273 0.1	3 kPa K ⁻¹ 0.8 kPa K ⁻¹
s 1 [33]	273 0.1	0.8 J kg ⁻¹ K ⁻¹ 0.02 kg ⁻¹ K ⁻¹
s 1 [34]	273 0.1	0.7 J kg ⁻¹ K ⁻¹ 0.5 J kg ⁻¹ K ⁻¹
c_p 61 [33]	16-268 0.1	relative 2% relative 0.8%
c_p 59 [35]	2-27 0.1	relative 2% relative 3%
v 10 [36]	93-263 0.1	1 cm ³ kg ⁻¹ 0.9 cm ³ kg ⁻¹
v 10 [21]	13-193 0.1	0.3 cm ³ kg ⁻¹ 0.5 cm ³ kg ⁻¹
v 1 [37]	273 0.1	0.8 cm ³ kg ⁻¹ 0.3 cm ³ kg ⁻¹
v 1 [38]	227 0.1	0.4 cm ³ kg ⁻¹ 1.1 cm ³ kg ⁻¹
v 28 [39]	268-270 0.1	0.2 cm ³ kg ⁻¹ 0.1 cm ³ kg ⁻¹
v 1 [40]	273 0.1	0.04 cm ³ kg ⁻¹ 0.09 cm ³ kg ⁻¹
v 34 [41]	20-273 0.1	0.5 cm ³ kg ⁻¹ 0.6 cm ³ kg ⁻¹
v 6 [7]	251-273 0.1-211	10 cm ³ kg ⁻¹ 12 cm ³ kg ⁻¹
v 5 [42]	238 0.1-201	1 cm ³ kg ⁻¹ 1.4 cm ³ kg ⁻¹
$(\partial v / \partial T)_P$ 7 [43]	243-273 0.1	2 mm ³ kg ⁻¹ K ⁻¹ 1.7 mm ³ kg ⁻¹ K ⁻¹
κ_s 15 [44]	133-273 0.1	4 TPa ⁻¹ 3.4 TPa ⁻¹
κ_s 6 [45]	60-110 0.1	1 TPa ⁻¹ 0.6 TPa ⁻¹
κ_s 1 [46]	253 0.1	8 TPa ⁻¹ 6.5 TPa ⁻¹
κ_s 3 [47]	257-270 0.1	0.7 TPa ⁻¹ 1.1 TPa ⁻¹
κ_s 7 [42]	238-268 0.1	0.7 TPa ⁻¹ 0.6 TPa ⁻¹
κ_s 5 [42]	238 0.1-201	0.7 TPa ⁻¹ 0.4 TPa ⁻¹
$(\partial \kappa_s / \partial P)_T$ 4 [46]	253-268 0.1	500 TPa ⁻² 600 TPa ⁻²

Table 4. Selected values for isothermal compressibility κ_T at the normal pressure melting point after different authors. *Values estimated from the curvature of the melting curve.

source	$\kappa_T / \text{TPa}^{-1}$
Bridgman [5]	360
Richards and Speyers [51]	120
Hobbs [12]	104
Wexler [13]	134
Yen [14], Yen et al. [17]	232
Henderson and Speedy [29] *	98
Wagner et al. [26] *	190
Tillner-Roth [25]	112
this paper	118

capacity at low temperatures as well as the pressure independence of residual entropy are intrinsic properties of the potential function.

Of all properties available from eq. (1), we discuss in the following only few which could significantly be improved by the actual formulation.

4. Isentropic Compressibility

Experimental data for isothermal compressibility κ_T at 0 °C and normal pressure vary between 360 TPa^{-1} and 100 TPa^{-1} (Table 4), and this significant uncertainty remains present in more recent reviews of ice properties [10, 17].

More reliable values are available for isentropic compressibility, κ_s , which can be computed from the elastic moduli of the ice lattice, determined acoustically or optically with high accuracy (see [4] for details). Computed from elastic constants, data for κ_s at normal pressure with errors from less than 1% up to 8% are reproduced by the current formulation within their uncertainties over the temperature interval 60-273 K (Fig. 1), as are high-pressure data at -35°C between 0.1 and 200 MPa.

5. Melting Curve

The melting pressure equation of Wagner et al. [26] describes the entire phase boundary between liquid water and ice Ih with an uncertainty of 3% in melting pressure. On the other hand, the freezing temperature of water and seawater derived by Feistel [1] is more accurate at low pressures but invalid at very high pressures. The formulation given in the actual paper takes the benefits of both formulae, i.e. it provides the most accurate melting point measurements of Henderson and Speedy [29]

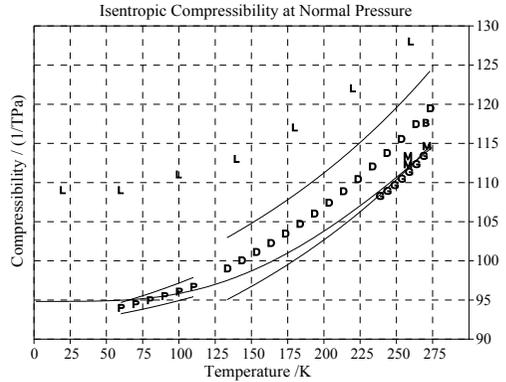


Fig. 1. Isentropic compressibilities at normal pressure as computed by eq. (4.1) are shown as a curve. D: data computed from the correlation functions for elastic moduli of Dantl [44] with about 3% error shown as lines above and below, P: correspondingly computed data of Proctor [45] with about 1% error bounds, L: [52], not used, B: [46], M: [47], G: [42].

with 50 mK mean deviation up to 150 MPa (Fig. 2).

Melting temperature T_f of ice at given pressure P is given by equal chemical potentials of the solid and the liquid phase,

$$g(T_f, P) = g_w(T_f, P), \quad (3)$$

using eq. (1) for ice and the IAPWS-95 formulation for water.

Ginnings and Corruccini [32] determined their calorimeter calibration factor K_{cal} to be

$$K_{\text{cal}} = \frac{L \cdot v_{\text{Hg}}}{v - v_{\text{W}}} = 270\,370 \pm 60 \text{ J kg}^{-1} \quad (4)$$

and used it for accurate determination of ice density $\rho = 1/v$ by means of melting enthalpy L , and specific volumes of water v_{W} and mercury v_{Hg} . This way the accuracy of ice density is limited by the uncertainty of L , namely 0.06%, while the even better accuracy of the calibration factor itself is 0.02%. The calibration factor is proportional to the Clausius-Clapeyron slope χ of the melting curve at normal pressure,

$$\begin{aligned} \chi &= -\frac{dT_f}{dP} = -\frac{v - v_{\text{W}}}{s - s_{\text{W}}} = \frac{T_m v_{\text{Hg}}}{K_{\text{cal}}} \\ &= 74.311 \pm 0.015 \text{ K GPa}^{-1}. \end{aligned} \quad (5)$$

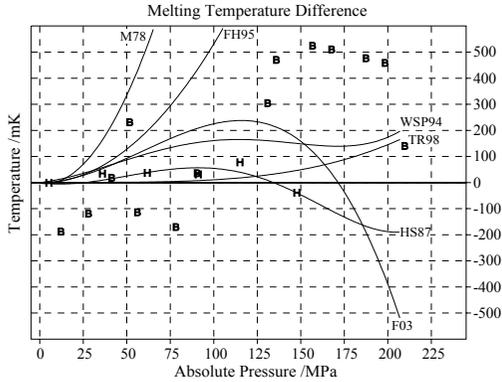


Fig. 2. Melting temperatures as functions of pressure in comparison to eq. (3) of this paper. Data points are: B: [5], H: [29]. Curves are labelled by M78: [53], FH95: [24], WSP94: [26], TR98: [25], HS87: [29], F03: [1]

The Gibbs function of this paper provides for melting point lowering the Clausius-Clapeyron coefficient $\chi = 74.305 \text{ K GPa}^{-1}$, which fits well into the 0.02% uncertainty interval of (5). Other standard formulae like that of Bridgman [7], $\chi = 73.21 \text{ K GPa}^{-1}$, the equation of Millero [53], $\chi = 75.3 \text{ K GPa}^{-1}$, or the one of Wagner et al. [26], $\chi = 72.62 \text{ K GPa}^{-1}$, are significantly beyond this error limit (5).

At normal pressure, eq. (3) provides the melting temperature $T_M = 273.152518 \text{ K}$. Making use of the fact that triple point temperature and normal pressure are exact by definition, and taking into account the small errors of the triple point pressure (Table 1) and of the Clausius-Clapeyron coefficient (5), the possible error of this normal melting temperature is estimated as only $2 \mu\text{K}$ [4].

6. Ice-Seawater Equilibrium

The phase equilibrium between seawater and ice is determined by equal chemical potentials of ice and water in seawater,

$$g(T_f, P) = g_{\text{sw}}(S, T_f, P) - S \cdot \left(\frac{\partial g_{\text{sw}}}{\partial S} \right)_{T, P} \quad (6)$$

$g_{\text{sw}}(S, T, P)$ is the Gibbs function of seawater [1]. Salinity S is the mass fraction of salt in the solution, which in oceanography is usually expressed in units of practical salinity (psu), which is about 1 g kg^{-1} . Equation (6) provides the freezing temperature T_f of

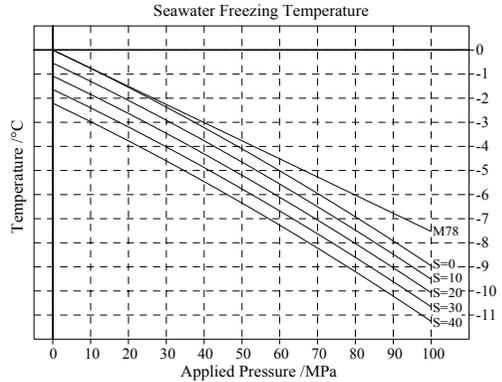


Fig 3. Freezing temperatures in $^{\circ}\text{C}$ of seawater with practical salinities S from 0 to 40 as indicated at the curves under applied pressure p (relative to normal pressure P_0) from 0 MPa to 100 MPa. M78 is Millero's formula for $S = 0$ [53].

seawater as function of salinity and pressure for the first time over the entire oceanic pressure range 0 - 100 MPa (Fig. 3). The accuracy can be estimated as 2 mK at normal pressure [1] and about 30 mK for higher pressures, as follows from the mean deviation from the measurements of Henderson and Speedy [29]. The main advantages of the new freezing point formula are the extended pressure range, the very accurate Clausius-Clapeyron slope (section 5) and the significantly improved compressibility (section 4) of ice.

The current international standard freezing point formula applied in oceanography is the one of Millero [53], considered to be valid up to 5 MPa, with a Clausius-Clapeyron slope about 70 times less accurate than the one proposed in this paper, and without any consideration of ice compressibility.

Sea ice is a mixed phase of ice with seawater intrusions ("brine pockets"). Brine salinity as function of pressure and temperature follows from the equilibrium condition (6). Thermodynamic properties of sea ice can be described with a suitable Gibbs function of sea ice, which is obtained by combining the Gibbs functions of ice and seawater [54].

7. Conclusions

A single Gibbs function of ice Ih is proposed which covers the entire region of its existence in the T-P space.

339 data points of 26 different groups of measurements are reproduced by the new formulation within their experimental uncertainty.

The formulation is expressed in ITS-90 and is consistent with the IAPWS-95 formulation of fluid water [3] and the Gibbs function of seawater [1].

The formulation obeys Debye's cubic law at low temperatures and pressure-independence of residual entropy.

The uncertainty in compressibility of previous formulae is reduced by 100 times.

The uncertainty in Clausius-Clapeyron slope at normal pressure of previous formulae is reduced by 100 times.

The absolute entropy of liquid water at the triple point is found as $3522 \pm 12 \text{ J kg}^{-1}\text{K}^{-1}$.

The melting temperature at normal pressure is found as $273.152\,518 \pm 0.000\,002 \text{ K}$.

A Gibbs function of sea ice and the freezing point of seawater are available up to 100 MPa.

References

- [1] R. Feistel, *Progr. Oceanogr.* **58**, 43 (2003)
- [2] D. R. Jackett, T. J. McDougall, R. Feistel, D. G. Wright and S. M. Griffies, *J. Atm. Ocean. Techn.* (2005), in press
- [3] W. Wagner and A. Pruß, *J. Phys. Chem. Ref. Data*, **31**, 387 (2002)
- [4] R. Feistel and W. Wagner, *J. Mar. Res.* **63** (2005), in press
- [5] P. W. Bridgman, *Proc. Am. Acad. Arts Sci.* **47**, 441 (1912)
- [6] P. W. Bridgman, *Proc. Am. Acad. Arts Sci.* **48**, 309 (1912)
- [7] P. W. Bridgman, *J. Chem. Phys.* **3**, 597 (1935)
- [8] P. W. Bridgman, *J. Chem. Phys.* **5**, 964 (1937)
- [9] E. R. Ponder, *Physics of Ice* (Pergamon Press, 1965)
- [10] N. E. Dorsey, *Properties of ordinary water-substance*. (Hafner Publishing Company, 1968)
- [11] N. H. Fletcher, *The Chemical Physics of Ice*. (Cambridge University Press, 1970)
- [12] P. V. Hobbs, *Ice Physics* (Clarendon Press, 1974)
- [13] A. Wexler, *J. Res. Nat. Bur. Stand.* **81A**, 5 (1977)
- [14] Y.-C. Yen, *CCREL Report 81-10*, 1 (1981)
- [15] R. W. Hyland and A. Wexler, *Trans. Am. Soc. Heat. Refrig. Air Cond. Eng.* **89**, 500 (1983)
- [16] O.V. Nagornov and V. E. Chizhov, *J. Appl. Mech. Techn. Phys.* **31**, 343 (1990)
- [17] Y.-C. Yen, K. C. Cheng and S. Fukusako, in *Proceedings 3rd International Symposium on Cold Regions Heat Transfer*, eds. J. P. Zarling and S. L. Faussett (Fairbanks, AL, 1991) p. 187
- [18] V. F. Petrenko, *CRREL Report 93-25*, 1 (1993)
- [19] V. F. Petrenko and R. W. Whitworth, *Physics of Ice* (Oxford University Press, 1999)
- [20] H. Eicken, in *Sea Ice*, eds. D. N. Thomas, G. S. Dieckmann (Blackwell Publishing, 2003), p. 22
- [21] R. Brill and A. Tippe, *Acta Cryst.* **23**, 343 (1967)
- [22] H. Tanaka, *J. Chem. Phys.* **108**, 4887 (1998)
- [23] K. Röttger, A. Endriss, J. Ihringer, S. Doyle and W. F. Kuhs, *Acta Crystall.* **B50**, 644 (1994)
- [24] R. Feistel and E. Hagen, *Progr. Oceanogr.* **36**, 249 (1995)
- [25] R. Tillner-Roth, *Fundamental Equations of State* (Shaker Verlag, 1998)
- [26] W. Wagner, A. Saul and A. Pruß, *J. Phys. Chem. Ref. Data*, **23**, 515 (1994)
- [27] L. Pauling, *J. Amer. Chem. Soc.* **57**, 2680 (1935)
- [28] J. F. Nagle, *J. Math. Phys.* **7**, 1484 (1966)
- [29] S. J. Henderson and R. J. Speedy, *J. Phys. Chem.* **91**, 3096 (1987)
- [30] G. Jancso, J. Pupezin and W. A. V. Hook, *J. Phys. Chem.* **74**, 2984 (1970)
- [31] D. R. Douslin and A. Osborn, *J. Sci. Instrum.* **42**, 369 (1965)
- [32] D. C. Ginnings and R. J. Corruccini, *J. Res. Natl. Bur. Stand.* **38**, 583 (1947)
- [33] W. F. Giauque and J. W. Stout, *J. Am. Chem. Soc.* **58**, 1144 (1936)
- [34] N. S. Osborne, *J. Res. Nat. Bur. Stand.*, **23**, 643 (1939)
- [35] P. Flubacher, A. J. Leadbetter and J. A. Morrison, *J. Chem. Phys.* **33**, 1751 (1960)
- [36] S. LaPlaca and B. Post, *Acta Cryst.* **13**, 503 (1960)
- [37] H. D. Megaw, *Nature*, **134**, 900 (1934)
- [38] F. K. Truby, *Science* **121**, 404 (1955)
- [39] T. R. Butkovich, *J. Glaciol.* **2**, 553 (1955)
- [40] G. Dantl and I. Gregora, *Naturwiss.* **55**, 176 (1968)
- [41] M. Jakob and S. Erk, *Mitt. Phys.-Techn. Reichsanst.* **35**, 302 (1929)
- [42] R. E. Gagnon, H. Kiefte, M. J. Clouter and E. Whalley, *J. Chem. Phys.* **89**, 4522 (1988)
- [43] T. R. Butkovich, *SIPRE Res. Rep.* **40**, 1 (1957)
- [44] G. Dantl, Dissertation (TH Stuttgart, 1967)
- [45] T. M. Proctor jr., *J. Acoust. Soc. Amer.* **39**, 972 (1966)
- [46] B. Brockamp and H. Rüter, *Z. Geophys.* **35**, 277 (1969)
- [47] P. H. Gammon, H. Kiefte and M. J. Clouter and W.W. Denner, *J. Glaciol.* **29**, 433 (1983)
- [48] L. A. Guildner, D. P. Johnson and F. E. Jones, *J. Res. Natl. Bur. Stand.* **80A**, 505 (1976)
- [49] *ISO STANDARDS HANDBOOK*. (International Organization of Standardization, 1993)
- [50] H. Preston-Thomas, *Metrologia* **27**, 3 (1990)
- [51] T. W. Richards, and C. L. Speyers, *J. Amer. Chem. Soc.* **36**, 491 (1914)
- [52] A. J. Leadbetter, *Proc. Roy. Soc. Lond.* **287**, 403 (1965)
- [53] F. J. Millero, *Unesco techn. pap. mar. sci.* **28**, 29 (1978)
- [54] R. Feistel, and E. Hagen, *Cold reg. sci. technol.* **28**, 83 (1998), corrig. **29**, 173 (1999)