Mathematical Model of the Laboratory Experiment that Simulates the Hydraulic Fracturing of Rocks under Supercritical Water Conditions

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Laboratory experiments of hydraulic injection in granite were carried out at super-high temperature and pressures environment. Fluid flow behavior was examined at temperatures up to 600 °C and confining pressures up to 100 MPa for various injection rates. The laboratory set-up capable to simulate the process of hydraulic fracturing of rock masses was used and tests were conducted for granite cylindrical specimens. After applying the confining pressure onto the cylindrical specimen, the temperature was elevated up to a predetermined value. Heating of the specimen was undertaken using an internal heater. Water at high pressure was injected into the borehole (drilled along the axis of symmetry of the specimen) at a constant flow rate. In order to utilize the experimentally obtained data for assessing the permeability of the rock specimen, a mathematical model of fluid flow within the experimental setup was derived. The model accounts for the radial Darcy flow in the rock specimen, elastic displacement of the silver jacket due to the pressure exerted by the water outflow from the outer wall of the specimen. The jacket displacement leads to appearing a very thin gap between the specimen and the jacket, through which the injected water is squeezed away from the specimen. Measuring the flow rate of injected water, confining pressure, and pressure in the borehole during the experiment, the effective permeability of the specimen was computed by the equation obtained from solution of the fluid dynamics model within the experimental set-up. This equation accounts for the water viscosity at the temperature and pressure conditions, for the Young modulus and Poisson ratio of the silver jacket, and for jacket thickness. The results show that under the high temperature-high pressure conditions, when water is in supercritical state, the permeability of the rock exhibits a sharp increase. The present experimental results suggest that peculiarities of the interaction of supercritical water and rocks can lead to generating a highly permeable micro-fracture network, which is very advantageous for further utilization of this stimulated supercritical rock masses as a perspective site for a geothermal power plant.

1. Introduction

Most Hot Dry/Wet Rock (HDR/HWR) development programs, and all geothermal projects presently completed for commercial energy utilization, have focused on water/vapor reservoir(s) of limited size and temperatures below the critical temperature of water.

To date, high-temperature rock masses have been identified by deep exploration drilling at Kakkonda, Japan [1], Larderello, Italy [2] and Nesjavellir, Iceland [3]. As exemplified by the field survey at the Kakkonda geothermal field in Japan, extremely high temperature (above the critical point of water) rock masses may be found just below existing geothermal reservoirs. It may be one of the useful options to exploit very high temperature rock masses underneath the existing geothermal reservoirs in order to enhance the geothermal energy extraction. To this end, the laboratory analysis of the water-rock interaction at super-high temperatures and pressures can be quite helpful for understanding the physical processes that take place in the rock masses at big depths. In this paper, such high temperature rock masses whose temperature and pressure conditions exceed the critical point of water are referred to as a supercritical rock mass. It is also suggested that no significant natural fracture system is expected to exist in such supercritical rock masses for a volcanic arc system. This may call for the creation of an artificial reservoir and water circulation system in order to exploit supercritical rock masses. It is proposed to utilize the discovered supercritical water-induced micro-cracking phenomenon in order to create an artificial reservoir in deep-seated rock masses whose temperature and pressure conditions exceed the critical point of water.

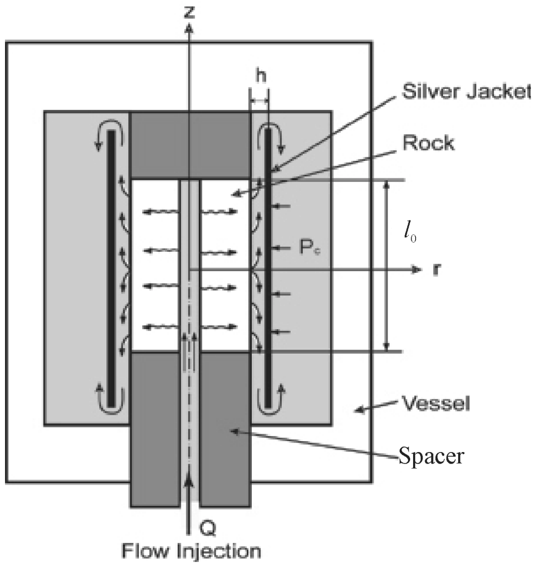

Fig. 1. Schematic illustration of a high temperature-pressure autoclave system used for the laboratory modeling of the process of hydraulic stimulation

2. Experimental Procedure

The rock used in this study was Iidate Granite, Takidani Granite, Kakkonda Granite, and Tsukuba Granite. Simulated hydraulic stimulation tests were conducted on the granite using thick-walled cylindrical specimens. The outer and inner diameters of the cylindrical specimens were 45 and 5 mm, respectively, and the specimen height was 90 mm. A schematic illustration of the hydraulic stimulation test apparatus used is given in Fig.1. The maximum temperature and confining pressure achievable with this apparatus are 600 °C and 150 MPa, respectively. The maximum axial load is 100ton and the corresponding maximum axial stress is approximately 625 MPa for the above-mentioned specimen dimensions. The cylindrical specimen and metallic spacers were inserted into a 0.3 mm thick silver jacket to form a specimen assembly, as shown in Fig. 1. The specimen assembly was then placed between the upper and lower pistons. Aluminum disks of 1 mm thickness were placed between the specimen and metallic spacer in order to prevent the leakage during the simulated hydraulic stimulation tests. After applying confining pressure onto the cylindrical specimen, the temperature was elevated at a constant rate of 3°C/min up to a predetermined value. The heating of the specimen was undertaken using an internal heater. The temperature control was conducted using a thermocouple placed in the middle height of the specimen, and the temperature distribution was monitored at the upper and lower positions. The borehole temperature was also measured using a thermocouple during the hydraulic stimulation tests. After the temperature reached the predetermined level, water was then injected into the borehole at a constant flow rate. The flow rates used in this study were in the range of 0.5 mm^3 /sec -300 mm^3 /sec.

3. Mathematical Model of Fluid Dynamics

In order to use the experimentally obtained data for assessing the rock permeability, mathematical model of the laboratory setup was developed. The model accounts for the radial Darcy flow in the rock specimen, elastic deformation of the silver jacket due to the pressure exerted by the water outflow from the outer wall of the specimen, which leads to creating a very thin gap between the specimen and jacket, through which the injected water is squeezed away from the specimen. Measuring the flow rate of injected water, Q, confining pressure, P_c, and pressure in the borehole, P_b, during the experiment, the equation for the effective permeability of the specimen, K, was obtained. This equation accounts for the length of the specimen, l_0, outer, r_0, and inner, r_b , radiuses of the specimen (i.e. borehole radius), for the water viscosity, μ_s, under high temperature-pressure (supercritical) conditions, for Young modulus, E, and Poisson ratio, σ, of the silver jacket, and for the silver jacket thickness, δ.

Attempt has been made to analyze the experimental data and to calculate the permeability of stimulated rock by the direct utilization of Darcy equation. One can suggest that based on Darcy equation the permeability of the stimulated rocks could be computed with the following formula:

$$K = \frac{\rho_w \mu_s Q}{2\pi l_0 \rho_s (P_b - P_0)} \ln\left(\frac{r_0}{r_b}\right), \tag{1}$$

where P_0 is pressure on the outer surface of spacemen (at $r=r_0$), ρ_w and ρ_s, are water densities at the injection point (at room temperature) and in the rock sample (supercritical conditions), respectively. Preliminary estimates of the stimulated rock permeability in supercritical conditions made with equation (1), where all parameters on the right-hand side were obtained experimentally (it was assumed that approximately $P_0 \approx P_c$), demonstrated that the value of permeability K, computed by equation (1) was significantly lower than the real permeability of the stimulated rock sample measured after the experiment (see Fig. 2). This discrepancy can be attributed to the fact that the effect of viscous drag of water flow in a very thin gap between the confining silver jacket and spacemen was ignored. The silver jacket around the specimen is pressed to the specimen surface by the force exerted by the confining pressure of fluid, P_c, surrounding the jacket in the vessel. After the fluid injecting in the borehole under the pressure $P_b > P_c$, the silver jacket exhibits a slight deformation in the radial direction, so that a thin gap between the spacemen and the jacket is formed. Thickness of this gap, h, can be readily computed from the Landau solution for the radial displacement of the cylindrical elastic shell due to the differences in pressures acting on the inner and outer surfaces of the shell [4]. This solution leads to the following equation:

$$h = \frac{(\overline{P} - P_c) r_0^2 (1 - \sigma^2)}{E\delta}, \tag{2}$$

where $\overline{P}$ is fluid pressure applied to the inner wall of the jacket (i.e. in the gap). All parameters on the right-hand side of equation (2) are given or measured over the experiment except the unknown pressure $\overline{P}$. This value can be obtained from the solution of the fluid dynamics problem in the gap between the specimen and silver jacket. Governing equations and boundary conditions on the outer surface of the spacemen, $r=r_0$, and on the inner surface of the silver jacket, $r = r_s = r_0 + h$, for this problem are following

$$\frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial z} = 0, \tag{3}$$

$$\frac{\partial P}{\partial z} = \mu_s \frac{\partial^2 v_z}{\partial r^2}, \tag{4}$$

$$r = r_0;\ v_z = 0,\ v_r = v_0; \tag{5}$$

$$r = r_0 + h;\ v_z = 0,\ v_r = 0; \tag{6}$$

where

$$v_0 = Q/(2\pi l_0 r_0). \tag{7}$$

Solution of the boundary-value problem (3)-(6) is rather straightforward and it can be readily shown that fluid pressure in the gap satisfies the following equation:

$$P = P_c + \frac{6\mu_s v_0 [z^2 - (l_0/2)^2]}{h^3}. \tag{8}$$

The latter equation leads to the following expression for the mean fluid pressure in the gap:

$$\overline{P} = \frac{2}{l_0} \int_0^{l_0/2} P dz = P_c + \frac{\mu_s l_0^2 v_0}{h^3}. \tag{9}$$

Accounting for equation (9), the formula for the gap thickness (2) can be converted to the following form:

$$h = \sqrt[4]{\frac{l_0^2 \mu v_0 r_0^2 (1 - \sigma^2)}{E\delta}}. \tag{10}$$

Substituting v_0 and h defined by equations (7) and (10) into equation (9) and assuming that $P_0 \approx \overline{P}$, leads to the final expression for the pressure on the external surface of the spacemen:

$$P_0 = P_c + \left(\frac{\mu_s l_0 Q}{2\pi r_0}\right)^{1/4} \left(\frac{\delta E}{r_0^2 (1 - \delta^2)}\right)^{3/4}. \tag{11}$$

Substituting this expression into the equation (1) yields the final equation for the rock permeability, which accounts for the viscous drag force in the gap and physical properties of the jacket.

4. Results and Discussion

The importance of correct application of formula (1) for calculation of permeability is illustrated in Fig. 2.

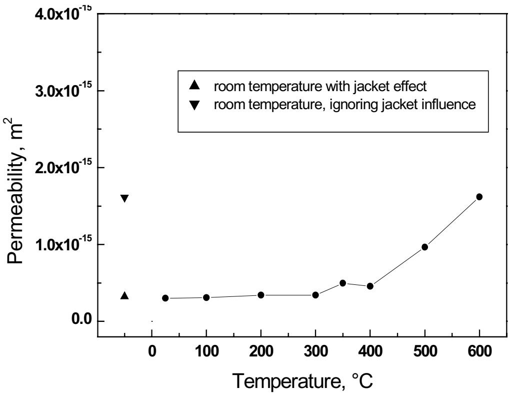

Fig. 2. Temperature dependence of the computed permeability for the hydraulically stimulated Iidate Granite.

Solid line connecting dots represents the experimental data obtained during hydro-fracturing of the Iidate Granite sample at different temperatures. The permeability of the stimulated in supercritical conditions granite was calculated after high temperature-pressure experiments at the room temperature and atmospheric pressure. The results computed by equations (1) and (11) are presented by turned over triangle totally coincide with permeability computed by the same equations at the final stage of the high pressure-temperature supercritical stimulation. Whereas the permeability of the same rock sample (indicated by the lower triangle) computed by formula (1), where it's assumed that $P_0 = P_c$, is much lower than the actual permeability of the rock. The latter leads to the evident conclusion that ignoring the influence of viscous drag in the gap and elastic properties of the jacket may lead to significant underestimation of the rock permeability.

In order to assess the effect supercritical hydraulic stimulation on different types of rocks, the series of laboratory experiments were performed for different samples of surface granites, namely, for Iidate Granite, Takidani Granite, and Tsukuba Granite. The results of these tests are presented in Fig. 3. It can be readily seen that the permeability increases rapidly when the temperature exceeds the value of 350 ī due to hydro-thermally induced micro-cracking in each of three types of granite. The growth of mean apertures of cracks should be the main reason for the increase of permeability.

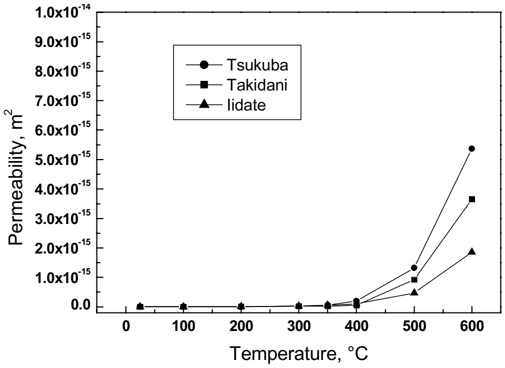

Fig.3. Variation of permeabilities of different surface rocks due to the temperature increase above supercritical point.

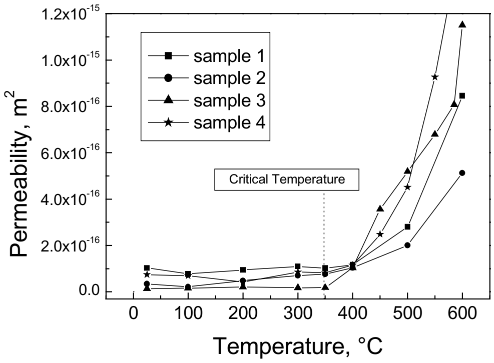

Fig. 4. Growth of permeability of the deep-seated rocks by hydraulic stimulation in supercritical conditions

It should be noted that the above discussed experiments and computations were carried out for surface granites. However, in a view of possible exploitation of deep-seated reservoirs for the geothermal energy production it is important to investigate peculiarities of the hydraulic stimulation of this type of rocks in supercritical conditions. The supercritical hydraulic stimulation has been carried out for 4 types of the deep-seated Kakkonda Granite. According to the injection tests, although the permeability increased rapidly but was far below those values that were achieved in the case of surface rock at the temperature above 350 ī . In contrast, the result of static (low flow rate) test showed that no micro-cracking occurred in those 4 types of Kakkonda granite even at 500 ī and 550 ī . Therefore, the flow test at high flow rate was carried out for generating cracks. The results of this high-flow-rate experiment presented in Fig. 4 suggest that thermal and mechanical stresses can induce the process of very intensive micro and macro cracking in deep-seated rocks, and therefore, creating high thermo-mechanical stress conditions can be very beneficial for developing the highly permeable deep-seated hot-dry-rock reservoirs.

Acknowledgements

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References

[1] K. Ikeuchi, R. Komatsu, N. Doi, Y. Sakagawa, M. Sasaki, H. Kamenosono & T. Uchida, *Geothermal Resources Council Transactions,* **20,** 609 (1996).

[2] G. Cappetti, C. Romano, U. Cigni, P. Squarci, G. Stefani & L. Taffi, *Proc. of GRC, Int. Symp. on Geothermal Energy,* 303 (1985).

[3] B. Steingrimsson, A. Gudmundsson, H. Franzson, and E. Gunnlaugsson, *Proc. 15th Workshop on Geothermal Reservoir Engineering, Stanford University*, **SGP-TR-130,** 81 (1990).

[4] L.D. Landau and E.M. Lifshiz, *Theory of elasticity*, Pergamon Press, Oxford, 1986.